Ground State Rotational Bands in ^{16}O, ^{40}Ca and ^{208}Pb?

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It is pointed out that a considerable amount of evidence exists to support the concept of a ground state rotational band based on tetrahedral deformation for ^{16}O, ^{40}Ca and ^{208}Pb. The possible meaning of this interpretation as well as other well-known alternatives are discussed.

Ground state rotational bands are familiar phenomena associated with the deformed nuclei. In even-even nuclei, for example, they are based on 0^+ ground states and consist of the members 2^+, 4^+, 6^+, ... which are spaced according to the $I(I+1)$ law. The transition strengths, from the 2^+ level to the ground state, yield information on the intrinsic quadrupole moments and in turn on the shape of the nuclei: large deformations are known in the rare-earth and actinide regions and essentially none for nuclei near magic numbers. This is particularly evident from the absence of the corresponding 2^+ level in these latter nuclei. For ^{16}O and ^{40}Ca the picture has been somewhat modified by supplementary experimental evidence such that these nuclei are now believed to have a non-spherical component in their ground state wave functions [1, 2].

For a non-spherical ground state a rotational band is expected to occur with intraband transition probabilities which are essentially determined by the amount of deviation from sphericity. The details of this band will depend on the moment of inertia and on intrinsic symmetries which may forbid several of the predicted states. In this note we want to point out that a considerable amount of evidence exists to support the concept of ground state rotational bands for the doubly magic nuclei ^{16}O, ^{40}Ca and ^{208}Pb. The appearance of these bands suggests a rather special departure from the spherical shape which has all the symmetries of the tetrahedral point symmetry group.

$$R\,\psi(r_1, \dots, r_A) = \psi(r_1, \dots, r_A) \quad \text{for all } R \in T_{\mathrm{d}}(24).$$

Investigations of the structure of a rotational band for a given intrinsic shape is done by means of group theoretical methods. For tetrahedral symmetry in particular the theory has been discussed by Jahn [3] in connection with molecules, and by Dennison [4] in applications to ^{16}O. As a result one expects a band consisting of the levels 0^+, 3^-, 4^+, 6^+, 6^-, ... which are spaced according to the $I(I+1)$ law.

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The experimental situation is visualized in Fig. 1, which shows the theoretical prediction together with the spectra of ^{16}O, ^{40}Ca and ^{208}Pb. The spins and parities of most of the levels are known yet have been omitted in the figure.

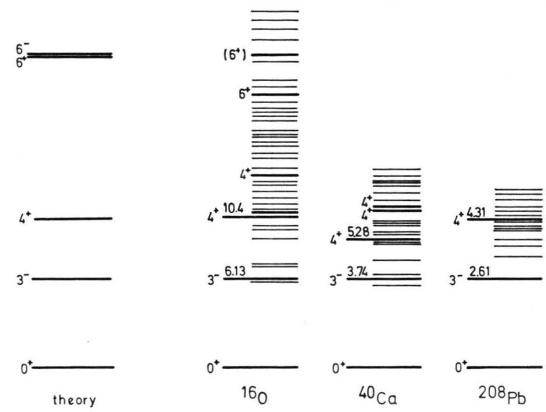

Fig. 1. The experimental low-energy spectra of ^{16}O, ^{40}Ca and ^{208}Pb are compared with the theoretical ground state rotational band for systems with intrinsic tetrahedral symmetry. The spectra have been scaled differently for each nucleus such that the 3^- levels coincide, and also to emphasize the states relevant for this interpretation. Other well-known alternatives are discussed in the text. The data are taken from Refs. [5—11].

The spectra have been scaled differently for each nucleus such that the 3^- levels coincide, and have been drawn to emphasize the states relevant for this interpretation. The heavy lines indicate the candidates for the rotational band assignment, the rest corresponds to levels which may be interpreted in terms of intrinsic nuclear vibrations. To the extent that these latter levels have spins and parities equal to any one of the rotational levels they have to be considered in this discussion. A quantitative estimate of their influence is possible only in the framework of a theory of nuclear vibrations and their interaction with the rotational levels, both can be given only within specific nuclear models. For ^{16}O such questions are dealt with by Dennison[4] in the α-particle model. For ^{40}Ca and ^{208}Pb similar discussions do not exist.

The discussion here is given essentially without reference to a theory of nuclear vibration and single particle exitations allowing for a more model independent comparison of the three nuclei.

The most outstanding feature in the low-energy spectrum of all three doubly magic nuclei is the strong collective 3^- level and the simultaneous absence of the typical 2^+ level which is known in essentially all other nuclei. (Within the model the absence of the 2^+ level is equivalent to the vanishing of the quadrupole moment when assuming intrinsic tetrahedral symmetry.) At the same time there exist 4^+ levels again in all the three nuclei with appropriate energies to fit the $I(I+1)$ law, and which are known to be strongly collective also. (In ^{208}Pb e.g. the 3^- and 4^+ ground state transitions are characterized by 40 and 15 Weisskopf units respectively[12]. B(E3) values for ^{16}O and ^{40}Ca are reported in Refs. 19, 20.) A more detailed discussion, especially with reference to other levels will be resumed subsequently for each nucleus individually.

^{16}O: The energy of the 4^+ level lies within 1% of the prediction of the $I(I+1)$ law when based on the 3^- energy. There exist further levels (3^-: 11.44 MeV, 11.63 MeV and 4^+: 11.09 MeV, 13.88 MeV) which may have considerable influence on the proposed rotational levels such that the 1% agreement presumably has to be considered accidental. At 21.2 MeV there exists a 6^+ level which fits the $I(I+1)$ law again with 1%.

The experimental situation in this case is not quite clear. The level has been reported by two groups[6, 7] yet has been put in brackets in the latest Ajzenberg-Selove[8] tables. There exist three more 6^+ levels at lower energies and the 19.34 MeV level may indeed be an alternative candidate. Measurements of the 6^+-4^+ intraband transitions will be needed in order to clarify this point.

It may be worth noting that an estimate of the moment of inertia according to the formula (2/5) MR^2 with $R=1.1\,A^{1/3}$ places the 3^- level right at the experimental energy. Although this may not be too significant in view of the well-known difficulties with moments of inertia in general, it does show, however, that the spacing within the band is of the expected order of magnitude.

^{40}Ca: Three 4^+ levels are known in the region of the predicted energy, two of which are 5% too high and a further level at 5.28 MeV some 15% too low. The latter may be the appropriate rotational level nevertheless, being shifted downward on account of rotation-vibration interaction with the other two levels. Also, this level has other properties similar to the 10.4 MeV level in ^{16}O.

^{208}Pb: Here, the situation is comparatively simple. A 4^+ and a 3^- level exist, their energies fulfill the $I(I+1)$ law to within 1%. In the absence of further levels of the same spins and parities this deviation may be interpreted as being due to the centrifugal force.

We turn to discuss the transition strengths. The qualitative statement of strong collective transitions in all three cases has to be supported by detailed calculations and depends on a model of the nuclear charge distribution. Phenomenologically, one may introduce a model which has 2 parameters (in addition to the usual radius and surface thickness parameters). These 2 parameters indicate the octupole and hexadecupole moments of the charge distribution. Clearly, the intensity of the transitions within the band is related to the degree of deviation from a spherical charge distribution: the band becomes weaker as the nuclei become more spherical, and it disappears completely in the limit of intrinsically spherical nuclei. The relative intensity of the transition from the 3^- and the 4^+ levels to the ground state is related to the extend to which a further symmetry element exists, which turns the tetrahedral symmetry into octahedral symmetry. For the latter the ground state rotational band consists of the states 0^+, 4^+, 6^+, ...; the 3^- level has completely disappeared. From these remarks it is clear that the measured $B(E\lambda)$ values, or also the elastic electron scattering formfactors[13–17] can be fitted quite accurately.

We now review some of the well-known alternative interpretations of the spectra of these nuclei. The negative parity states in ^{16}O and ^{40}Ca have been calculated by Gillet and Melkanoff[20] in RPA. In these calculations a 3^- level has emerged in both nuclei with energies and transition strengths which fit the experimental data quite well. For the positive parity states in these nuclei there exists the microscopic theory of Brown and Green[1] and the more phenomenological model of Goldhammer and Prosser[2, 21]. According to these models the first excited 0^+ level is considered the strongly deformed orthogonal mixture to the ground state, and has along with it a rotational band $0^+, 2^+, 4^+, 6^+, \ldots$[22].

In ^{16}O there exists a further 0^+ level at 11.25 MeV, again with a rotational band of the same form. Experimentally the assignment of these bands have been mainly supported by the intraband $B(E2)$ values. The details may be found in Refs.[9, 22, 23]. For ^{208}Pb the situation is somewhat different, partly because theoretical microscopic calculations are more difficult to perform for this heavy nucleus. Discussions are given by Gillet and Sanderson[24] and by Ring and Speth[25].

We now discuss briefly the possible meaning of such ground state rotational bands. For ^{16}O (and possibly also for ^{40}Ca) the result can be quite naturally understood in terms of the α-particle model[4, 26, 27] and its modifications. A different interpretation which may apply for ^{208}Pb also depends on the assumption of tetrahedral symmetry of the usual self-consistent potential. For ^{16}O a discussion based on this assumption has been given by Onishi and Sheline[28].

Concluding, we summarize the evidence in support of ground state rotational bands in the three doubly magic nuclei ^{16}O, ^{40}Ca and ^{208}Pb:

(i) there exist 3^- levels in the low-energy spectrum of these nuclei, and at the same time the 2^+ levels which are found in essentially all other nuclei are absent;
(ii) at about the right energy to fulfill the $I(I+1)$ law there exist 4^+ levels, and
(iii) all $3^- \rightarrow 0^+$ and $4^+ \rightarrow 0^+$ ground state transitions are known to be strongly collective ,in agreement with the theoretical expection for rotational levels.

The assumption of these ground state rotational bands may be further tested by measuring the intraband transition probabilities, particularly the $B(E3)$ value for the $4^+ \rightarrow 3^-$ transitions, and also by searching for the predicted 6^- level in ^{16}O, and both the 6^+ and 6^- levels in ^{40}Ca and ^{208}Pb[29].

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